Ion drag force in complex plasmas

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The problem of calculating the ion drag force in complex plasmas is considered. It is shown that the standard theory of Coulomb scattering usually fails for the ion-dust elastic collisions. A simple approach to extend this theory is proposed. This leads to a considerable enhancement in the ion-dust elastic scattering cross section and, hence, increases the ion drag force in comparison with the previous analytical results. Analysis shows that the ion drag usually exceeds the electrostatic force in the limit of weak electric field. We suggest that this is the cause of the central ''void'' observed in microgravity complex plasma experiments.

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Complex or dusty plasmas are multicomponent plasmas whose components are micron-sized dust particles (grains), electrons, ions, and neutral atoms (molecules). Since the dust component can be visualized and analyzed at the kinetic level, complex plasmas are recognized as valuable model systems for the study of phase transitions [1–5], wave phenomena [6,7] and other collective processes. In rf and dc glow discharges the grains are charged (negatively) due to collection of electrons and ions from a plasma. The electric field affects charged grains in two ways. First, it exerts an electrostatic force in the direction opposite to the field. The second effect is indirect: The momentum transfer from the positive ion current which is driven by the electric field causes a so-called ion drag force. This force is pointed along the field. The competition between these two forces often determines the grain location in the discharge chamber [5], it also strongly affects the properties of low-frequency waves in complex plasmas [7] and the interaction between grains [8,9].

The ion drag force F_I consists of two parts often referred to as *collection* and *orbital forces*. The collection force is associated with momentum transfer from the ions that are collected by the grain, while the orbital force is due to the momentum transfer from the ions that are scattered in the electric field of the grain (but not collected). The calculation of $F_{\rm I}$ has been addressed recently in several works [10–12]. Barnes *et al.* [10] modified the standard theory of pair collisions of charged particles in plasmas by taking into account the finite grain size and ion collection by the grain. The analytical expression obtained in Ref. [10] is widely used in the literature. A numerical calculation of the momentum-transfer cross section for elastic ion scattering was reported by Kilgore *et al.* [11] for a pointlike grain, with the potential distribution derived from a self-consistent numerical solution of the Poisson-Vlasov equation [13]. The obtained cross section was then used to determine the orbital component of the ion drag force and to study transport of small (submicron) dust grains in glow discharges [12].

A critical examination of the existing results on the ion drag force shows the inconsistencies between them. The analytical expression derived for arbitrary grain size [10] underestimates the numerical result of Ref. [11] by as much as one order of magnitude in the limit of a pointlike grain. On the other hand, the cross section obtained in Ref. [11] is not directly applicable for micron-sized grains typical for complex plasmas experiments.

In this paper, we propose a simple approach which can be used to estimate the ion drag force on an ''isolated'' dust grain in a low-pressure plasma. The obtained analytical expression is applicable for typical experimental conditions and shows reasonable agreement with earlier numerical results [11,14]. Using our results we compare the magnitudes of the ion drag and electrostatic forces in the limit of weak electric field and show that the ion drag usually dominates.

We consider a negatively charged grain at rest and assume that the inequalities $a \ll \lambda_{\rm D} \ll l_i$ and $\Delta \gg \lambda_{\rm D}$ are satisfied. Here a is the grain radius, $\lambda_{\rm D}$ is the screening (Debye) length of the plasma, l_i is the ion mean free path, and Δ is the intergrain distance. The positive ions are singly charged. The general expression for the ion drag force is

$$\mathbf{F_I} = m \int \mathbf{v} v f_i(\mathbf{v}) [\sigma_{\rm c}(v) + \sigma_{\rm s}(v)] d\mathbf{v}, \tag{1}$$

where $\mathbf{v}$ is the ion velocity and m is the mass, $f_i(\mathbf{v})$ is the ion velocity distribution function, and $\sigma_{\rm c}(v)$ and $\sigma_{\rm s}(v)$ are the (velocity dependent) momentum-transfer cross sections for the ion collection and scattering, respectively.

Collection cross section. If we assume there is no potential barrier for the ions moving towards the grain, the conservation of angular momentum and energy can be used to obtain the collection cross section. This approach, known as the orbital motion limited (OML) theory gives the maximum impact parameter at which ions are collected by a grain,

$$\rho_{\rm c} = a(1 + 2\rho_0/a)^{1/2}, \tag{2}$$

where $\rho_0(v) = Ze^2/mv^2$ is the Coulomb (Landau) radius ($Z > 0$ is the grain charge number). This parameter characterizes the scattering in the Coulomb field of the pointlike particle: Ions are deflected strongly if the impact parameter ρ is less than ρ_0 in the opposite case the scattering angle is small. Using Eq. (2) we get the collection cross section,

$$\sigma_{\rm c}(v) = \pi a^2 (1 + 2\rho_0/a). \tag{3}$$

Note that in the OML approach $\sigma_{\rm c}$ does not depend on the profile of the electrostatic potential around the grain.

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Orbital (scattering) cross section. To determine the orbital cross section σ_s it is necessary to know the exact distribution of the potential around the grain. This distribution is quite complicated. Within a few Debye lengths from the grain it can be well represented by a screened Coulomb potential, $U(r) \propto \exp(-r/\lambda_D)/r$, according to numerical simulations [13] and recent experimental data [15]. At larger distances, due to plasma absorption on the grain surface the potential has a different asymptote, $U(r) \propto r^{-2}$ [8,13]. Therefore, the precise determination of σ_s can be done only numerically. Such numerical calculation was reported in Ref. [11] for the case of a pointlike grain. It is indeed in good agreement with the earlier numerical results obtained for an attractive screened Coulomb potential [14] as illustrated below.

In order to obtain an analytical expression for σ_s it is usually assumed that only those ions approaching sufficiently close to the grain can contribute to the momentum transfer. For these ions the interaction potential is assumed to be of unscreened Coulomb form. The corresponding orbital cross section is

$$\sigma_s(v) = 4\pi \int_{\rho_{\min}}^{\rho_{\max}} \frac{\rho d\rho}{1+(\rho/\rho_0)^2} = 4\pi\rho_0^2 \Gamma, \tag{4}$$

where Γ is the Coulomb logarithm,

$$\Gamma(v) = \ln\left[\frac{\rho_0^2(v)+\rho_{\max}^2(v)}{\rho_0^2(v)+\rho_{\min}^2(v)}\right]^{1/2}. \tag{5}$$

The integration should start from the impact parameter at which ions are no longer absorbed by the grain: $\rho_{\min} = \rho_c$ [Eq. (2)]. The choice of $\rho_{\max}$ in standard Coulomb scattering theory is [10,11]

$$\rho_{\max} = \lambda_D. \tag{6}$$

This implies that due to screening ions with impact parameter larger than the Debye length practically do not "feel" the grain field and their contribution to the momentum exchange is negligible. This is true if the Coulomb radius is much smaller than the Debye length, i.e., when the parameter

$$\beta(v) = \rho_0(v)/\lambda_D. \tag{7}$$

is much less than unity, because the ratio of momentum transfer due to ions with $\rho < \lambda_D$ to the momentum transfer due to ions with $\rho > \lambda_D$ is proportional to $\ln(1/\beta)$ in this case. Therefore, to within logarithmic accuracy, it is sufficient to consider ions with impact parameters below λ_D. Note, that β is the only parameter which describes the scattering in the screened Coulomb potential, in the sense that the dependence of the scattering angle on the impact parameter is fixed for any given β.

While the standard Coulomb scattering theory works well for elastic collisions in usual electron-ion (weakly-coupled) plasmas it might, however, fail for the ion-grain collisions in complex plasmas. This can be illustrated by comparing the parameter β [Eq. (7)] for ion-ion and ion-grain collisions. In the former ($Z=1$) with ion thermal velocity, $v_{T_i} = \sqrt{T_i/m}$, we have $\beta(v_{T_i}) = \rho_0(v_{T_i})/\lambda_D \sim N_D^{-1} \ll 1$, where $N_D = n_i \lambda_D^3 \gg 1$ is the number of ions inside the Debye sphere. However, for complex plasmas the grain charge is large; $\beta(v_{T_i}) = z\tau a/\lambda_D$, where $\tau = T_e/T_i$ is the electron-to-ion temperature ratio and $z = Ze^2/aT_e$ is the dimensionless grain potential in units of T_e/e. Typically, in gas discharge plasmas $\tau \sim 10-100$, $a/\lambda_D \sim 10^{-1}-10^{-2}$ (for micron size grains), and z is always "of a few," so that $\beta(v_{T_i}) \sim 0.3-30$. Since β is proportional to a, for larger grains it can be even higher. Therefore, the range of the ion-grain interaction usually *exceeds* the Debye length. This is further illustrated by Fig. 1 which shows the distance of the ion closest approach to the grain r_0 during the collision as a function of the impact parameter ρ. We see that the ions with high β can enter the Debye sphere around the grain even if the impact parameter is considerably larger than the Debye length. Therefore, if the cutoff (6) is used in this case, then a significant fraction of the ion momentum transfer is neglected.

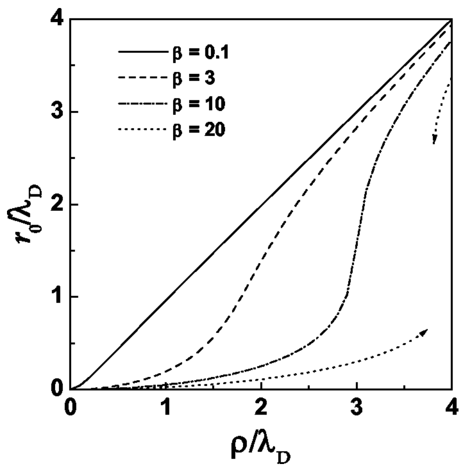

FIG. 1. Normalized distance of the ion closest approach to the grain during collision, r_0, vs the impact parameter ρ. The curves are calculated for a screened Coulomb potential. When the ratio of the Coulomb radius ρ_0 to the plasma Debye length λ_D exceeds the critical value ($\beta \equiv \rho_0/\lambda_D \geqslant 13.2$) a discontinuity appears due to a potential barrier for the ions moving towards the grain (see text).

We propose to improve the evaluation of the orbital cross section by taking into account the ions with impact parameters above the Debye length. In order to obtain analytical results we keep the approximation of the unscreened Coulomb potential leading to the cross section (4). But the determination of $\rho_{\max}$ in the Coulomb logarithm (5) is revised. We take into account the ions that *approach* the grain closer than λ_D. The definition of $\rho_{\max}$ is then

$$r_0(\rho_{\max}) = \lambda_D. \tag{8}$$

When β is large, condition (8) allows us to include ions with $\rho \gtrsim \lambda_D$ which are strongly deflected and approach the grain to $r_0 \lesssim \lambda_D$. The deflection is weaker for the ions that do not enter the Debye sphere; therefore, their contribution to σ_s is relatively small and is neglected. Thus, condition (8) should give more accurate results for the case $\beta \gtrsim 1$.

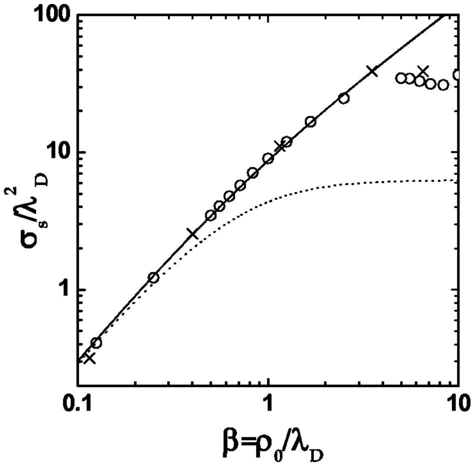

FIG. 2. Orbital cross section of ion-grain collisions σ_s [Eq. (4)] normalized to the squared Debye length λ_D^2 vs the parameter $\beta=\rho_0/\lambda_D$. The calculations are for the standard Coulomb logarithm (5) with cutoff (6) (dotted line) and for the modified Coulomb logarithm (10) (solid line). Crosses denote the self-consistent numerical calculation of Kilgore *et al.* [11]; open circles are the numerical results of Hahn *et al.* [14] obtained for an attractive screened Coulomb potential.

Assuming again no potential barrier for ions we obtain from Eq. (8)

$$\rho_{\max}=\lambda_D(1+2\beta)^{1/2}. \tag{9}$$

Note that in the limit $\beta\ll 1$ this condition reduces to the standard cutoff (6), as expected. Substituting $\rho_{\min}=\rho_c$ from Eq. (2) and $\rho_{\max}$ from Eq. (9) into Eq. (5) leads to particularly simple expression for the *modified* Coulomb logarithm,

$$\Gamma(v)=\ln\left[\frac{\rho_0(v)+\lambda_D}{\rho_0(v)+a}\right]. \tag{10}$$

In the limit of a pointlike grain the modified Coulomb logarithm (10) reduces to $\Gamma=\ln(1+1/\beta)$, whilst $\Gamma=\frac{1}{2}\ln(1+1/\beta^2)$ if the standard cutoff (6) is used. We can see that these two expressions are equivalent for $\beta\ll 1$, but have different asymptotics at $\beta\gtrsim 1$. To check the accuracy of the proposed approach we compare our results with the available numerical results [11,14]. The cross section (4) with Γ from Eq. (10), as well as with Γ corresponding to the cutoff (6) are plotted in Fig. 2, along with the numerical data. One can see that the simple approach proposed in this paper precisely describes these numerical results up to $\beta\sim 5$, whereas the standard cutoff (6) underestimated the cross section significantly above $\beta\sim 0.1$. However, for very large β ($\gtrsim 10$) our approach is also not adequate, because the use of the unscreened Coulomb potential is not justified any more.

It is useful to compare the collection cross section (3) and the orbital cross section (4) [with Γ from Eq. (10)] for the case of finite grain sizes. Figure 3 shows that the elastic scattering always dominates (as long as the proposed approach works). In contrast, in the model of Ref. [10] the contribution of the elastic scattering vanishes for $\rho_c(v_{T_i})>\lambda_D$.

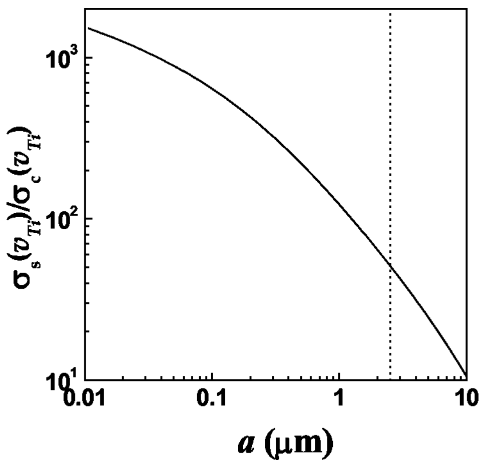

FIG. 3. Ratio of the orbital to the collection cross sections of ion-grain collisions (estimated for ion thermal velocity), $\sigma_s(v_{T_i})/\sigma_c(v_{T_i})$, vs grain radius a in μm (solid line). The calculation is for typical bulk plasma parameters: Ar gas, $T_e=1$ eV, $T_i=0.025$ eV, $n_i\simeq n_e=10^9$ cm^{-3}. The vertical dotted line at $a\simeq 2.5$ μm marks the point where $\rho_c(v_{T_i})=\lambda_D$. Note that for the parameters taken $\beta(v_{T_i})\simeq 2.9a_{\mu m}$.

Ion drag force. We integrate Eq. (1) for a given ion velocity distribution function to obtain an expression for the ion drag force. There are two limiting cases: superthermal ions ($u\gg v_{T_i}$) and subthermal ions ($u\ll v_{T_i}$), where u is the ion flow velocity. In the former case ions can be considered as monoenergetic and the integration can be replaced by the substitution of the flow velocity u into corresponding cross sections. Note that for the supersonic flow (e.g., in the rf electrode sheath) $u\gtrsim\sqrt{\tau}v_{T_i}$ and the Debye length is determined by electrons rather than by ions, $\lambda_D\sim\sqrt{\tau}\lambda_{Di}$. Then $\beta(u)\sim\beta(v_{T_i})/\tau^{3/2}\lesssim 1$, and the standard formula from Ref. [10] can be used to calculate the ion drag force.

For subthermal flow it is quite reasonable to use a shifted Maxwellian distribution, $f(\mathbf{v})\simeq f_0(v)(1+\mathbf{uv}/v_{T_i}^2)$, where $f_0(v)$ is an isotropic Maxwellian function. The integration of Eq. (1) with cross sections (3) and (4) gives

$$F_I=\frac{8\sqrt{2\pi}}{3}a^2 n_i m v_{T_i} u\left[1+\frac{\rho_0(v_{T_i})}{2a}+\frac{\rho_0^2(v_{T_i})}{4a^2}\Lambda\right], \tag{11}$$

where Λ is the Coulomb logarithm (10) integrated over $f(\mathbf{v})$,

$$\Lambda=2\int_0^\infty e^{-x}\ln\left[\frac{2\lambda_D x+\rho_0(v_{T_i})}{2ax+\rho_0(v_{T_i})}\right]dx. \tag{12}$$

The integration region is determined by the condition $\rho_{\min}(v)<\rho_{\max}(v)$, which is satisfied for all ion velocities if Eq. (9) is used to determine $\rho_{\max}$ (since $\lambda_D\lesssim a$). Therefore, the integration in Eq. (12) is performed from zero to infinity. Note that if we would use the standard cutoff (6), the inequality $\rho_{\min}(v)<\lambda_D$ sets up a lower limit for the integration [9].

Let us analyze expressions (11) and (12). First, we note that $\rho_0\to 0$ in the limit of an uncharged grain, and Eq. (11) recovers the well known result for the neutral drag force.

Next, let us compare Eq. (11) with the previously known expression [10]. The first two terms in square brackets of Eq. (11) represent the collection part of the force while the third one corresponds to the orbital part. The expression of Ref. [10] overestimates the collection force by the factor $3\pi/8 \simeq 1.18$ [since $\rho_0(v_{T_i})/a = z\tau \gg 1$]. This is because in Ref. [10] the integration over the ion velocity distribution function was replaced by the substitution of the average ion velocity $\sqrt{8T_i/\pi m}$ into the collection cross section (3). More important is that the expression for the orbital part obtained here is quite different from that in Ref. [10] for $\beta \gtrsim 1$. As was pointed out above, this is because in complex plasmas the range of ion-grain interaction is large and therefore, the standard Coulomb scattering theory is not applicable. For example, in a special case $\rho_c(v_{T_i}) \simeq \lambda_D$ the present approach gives for F_I the result ~ 25 times higher than that of Ref. [10] for plasma parameters of Fig. 3. This large difference is due to that the orbital part was neglected in Ref. [10] for $\rho_c(v_{T_i}) > \lambda_D$, whilst it still dominate over the collection part, as discussed above. In the opposite limit $\beta \ll 1$ the Coulomb logarithm (10) reduces to the standard form [with the cutoff (6)] and our expression for the ion drag force [Eqs. (11) and (12)] coincides with that from Ref. [10] (except for a numerical factor of the order of unity).

Comparison of electrostatic and ion drag forces. Let us compare the magnitudes of the electrostatic and the ion drag forces in the limit of a weak electric field E, when the ion drift is subthermal (this requires $E \ll T_i/el_i$). The ion drift velocity is given by $u = \mu_i E$, where $\mu_i = el_i v_{T_i}/T_i$ is the ion mobility. The electrostatic force is $F_E = -ZeE$. Since both forces $\propto E$ their ratio is a universal quantity, $|F_I/F_E| \simeq \delta l_i/\lambda_D$, where $\delta = (1/3\sqrt{2\pi})\beta(v_{T_i})\Lambda$ is a slowly increasing function of $\beta(v_{T_i})$, ranging from ~ 0.3 to ~ 0.5 for $1 < \beta(v_{T_i}) < 10$ (here we neglect small collection part and assume $\lambda_D \simeq \lambda_{Di}$). Our results were derived for the "collisionless" limit, when the ion mean free path l_i exceeds the range of the ion-grain interaction ρ_0 which, in turn, exceeds λ_D. Hence, in the limit of weak electric fields the ion drag is stronger than the electrostatic force for micron-size grains. This conclusion leads to a more physical insight into the mechanism of a void (dust-free region in the central part of a rf discharge) formation in complex plasma experiments under microgravity conditions. The electric field is weak in the center, and the ion drag (which is pointed outward) exceeds the electrostatic force (which is pointed to the center). The individual grains are pushed out of the center, leaving a void—as observed. In previous interpretations the ion drag was significantly underestimated, and hence other effects were invoked, e.g., thermophoresis, in order to explain the void formation [5,16].

The results presented in this paper might also be important for a variety of problems in complex plasmas. The appearance of wave modes and instabilities caused by the ion drag is expected [7]. The ion drag can lead to long-range attraction between dust grains and a negatively biased object (wire) [17], as well as between grains themselves [8,9]. Here the correct estimation of the ion drag force is required.

Limitations of the approach. The radial motion of ions during the collisions with a grain is described in terms of an effective potential energy, which has a potential barrier for $\beta > \beta_{cr} \simeq 13.2$ (for a screened Coulomb potential) [11]. This causes a discontinuity in the ion closest approach as illustrated in Fig. 1. Therefore, when $\beta(v_{T_i})$ becomes comparable with β_{cr}, most of the ions cannot overcome the potential barrier, and the proposed approach is no longer valid in the present form (both cross sections are overestimated). However, since we used the *unscreened* Coulomb potential to evaluate the orbital cross section (which is not justified for very large β) our results are applicable only for $\beta \lesssim 5$ (see Fig. 2). Hence, the potential barrier lies outside the range of validity of the present theory. The case $\beta > \beta_{cr}$ requires numerical analysis and is a subject of our future work. In addition, the effects of ion-neutral collisions and high dust density are neglected in our approach.

In conclusion, we have shown that the previous analytical approaches to estimate the ion drag force in complex plasmas were not adequate. We propose a simple procedure to improve the evaluation of the orbital part of the ion drag, which is justified by comparison with the earlier numerical results. This evaluation is valid over a wider range of parameters typical for complex plasma experiments and gives significantly larger magnitude of the ion drag force than previous analytical estimations for micron-size grains. This might be quite important for understanding of some basic processes in complex plasmas, e.g., the void formation, wave propagation, long-range interactions, etc.

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